Time in quantum gravity and black-hole information paradox

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Abstract

The fact that canonical quantum gravity does not possess a fundamental notion of time implies that the theory is unitary in a trivial sense. At the fundamental level, this trivial unitarity leaves no room for a black-hole information loss. Yet, a phenomenological loss of information may appear when some matter degrees of freedom are reinterpreted as a clock-time. This explains how both fundamental unitarity and phenomenological information loss may peacefully coexist, which offers a resolution of the black-hole information paradox.

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1 Introduction

The black-hole information paradox [1] (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10] for reviews) is one of the greatest unsolved puzzles in theoretical physics. One of the proposed solutions is a generalized quantum theory [11, 12, 13, 14] in which a pure quantum state describes information in the whole spacetime, rather than that on a spacelike hypersurface as in the usual formulation of quantum theory. Such a generalized state describes correlations between outgoing Hawking particles remaining after the complete black-hole evaporation and ingoing Hawking particles existing before that, thus avoiding the information loss.

Such a generalized quantum theory itself is motivated by a requirement of explicit spacetime covariance (see also [15, 16]). Yet, in this paper we show that explicit

spacetime covariance and generalized quantum theory are not essential at all. We explain how a similar idea works in a more conventional canonical quantum gravity. The essential ingredient common to both the approach in [13, 14] and that in the present paper is the idea that there is no fundamental notion of time evolution. In both approaches time is treated as a local quantum observable, and not as a global external parameter. We argue that such a treatment of time is sufficient to avoid information loss through the correlations between "future" and "past".

The paper is organized as follows. Sec. 2 is a brief review of some well-established aspects of the problem of time in quantum gravity, with an emphasis on the idea that time emerges as a clock-time described by the matter degrees of freedom. Sec. 3 is a novel application of that old idea, where we argue that such an emergent time in quantum gravity provides a natural resolution of the black-hole information paradox. Finally, the conclusions are drawn in Sec. 4.

2 Time in quantum gravity

Canonical quantum gravity is based on the Hamiltonian constraint

$$\mathcal{H}\Psi[g,\phi] = 0, \tag{1}$$

where $\mathcal{H}$ is the Hamiltonian-density operator and $\Psi[g,\phi]$ is the wave function of the universe, depending on gravitational and matter degrees of freedom denoted by g and ϕ, respectively. (On the technical level, the most promising variant of (1) is based on loop quantum gravity [19], where g denotes the loop variables.) Clearly, $\Psi[g,\phi]$ does not depend on an external time parameter, which is often referred to as problem of time in quantum gravity (see, e.g., [17, 18] for older reviews and [19] for a review written from a more modern perspective). Obviously, since $\Psi[g,\phi]$ does not depend on time, the information encoded in $\Psi[g,\phi]$ cannot depend on time either, i.e., information cannot be "lost". The lack of time dependence can be thought of as "time evolution" described by a trivial unitary operator

$$U(t) \equiv 1, \tag{2}$$

which means that the theory is unitary in a trivial sense. The quantity

$$\rho[g,\phi] = \Psi^*[g,\phi]\Psi[g,\phi] \tag{3}$$

can be interpreted as probability of given values g and ϕ, provided that $\Psi[g,\phi]$ is normalized such that

$$\int \mathcal{D}g\,\mathcal{D}\phi\,\Psi^*[g,\phi]\Psi[g,\phi] = 1. \tag{4}$$

In loop quantum gravity, the formal measure $\mathcal{D}g$ is mathematically well defined, and there are justified expectations that $\mathcal{D}\phi$ could be well defined too.

Even though there is no fundamental notion of time, a phenomenological notion of time can still be introduced. The most physical way to do it is to introduce a clock-time [19]. Essentially, this means that some of the matter degrees of freedom

describe the reading of a "clock". In this case the Hamiltonian $H = \int d^3x\,\mathcal{H}$ can be split as

$$H = \tilde{H} + H_{\text{clock}}, \tag{5}$$

where H_{clock} describes the clock and $\tilde{H}$ is the rest of the Hamiltonian. The Hamiltonian for a good clock can be approximated by a Hamiltonian of the form

$$H_{\text{clock}} \simeq \lambda P_{\text{clock}}, \tag{6}$$

where $\phi \equiv \{\tilde{\phi}, Q_{\text{clock}}\}$, Q_{clock} is the configuration variable representing the reading of the clock, P_{clock} is the canonical momentum conjugated to Q_{clock}, and λ is a coupling constant. Indeed, the resulting classical equation of motion

$$\frac{dQ_{\text{clock}}}{dt} = \frac{\partial H_{\text{clock}}}{\partial P_{\text{clock}}} \simeq \lambda \tag{7}$$

implies

$$Q_{\text{clock}}(t) \simeq \lambda t, \tag{8}$$

so Q_{clock} increases approximately linearly with time, which means that the value of Q_{clock} is a good measure of time.

In quantum theory the momentum P_{clock} is the derivative operator

$$P_{\text{clock}} = -i\hbar \frac{\partial}{\partial Q_{\text{clock}}}, \tag{9}$$

so (6) can be written as

$$H_{\text{clock}} \simeq -i\hbar \frac{\partial}{\partial q_{\text{clock}}}, \tag{10}$$

where $q_{\text{clock}} \equiv \lambda^{-1} Q_{\text{clock}}$. In this way, (1) implies a Schrodinger-like equation

$$\tilde{H}\Psi[g, \tilde{\phi}, q_{\text{clock}}] \simeq i\hbar \frac{\partial}{\partial q_{\text{clock}}} \Psi[g, \tilde{\phi}, q_{\text{clock}}]. \tag{11}$$

Even though (11) has the same form as the usual Schrödinger equation, we stress two important differences with respect to the usual interpretation of time in the Schrödinger equation. First, q_{clock} is a *quantum* observable, not a classical external parameter. Second, in most cases q_{clock} is a *local* quantity, not a quantity that can be associated with a whole spacelike hypersurface. As we shall see, these two features are essential for our resolution of the black-hole information paradox.

3 Implications on black-hole information paradox

Now assume that $\Psi[g, \phi]$ is a solution of (1) that describes an evaporating black hole. Of course, an explicit construction of such a solution is prohibitively difficult. Yet, under reasonable assumptions justified by understanding of semiclassical black holes, some qualitative features of such a hypothetic solution can easily be guessed without an explicit solution at hand. In particular, it is reasonable to assume that, at least

approximately, the degrees of freedom can be split into inside and outside degrees of freedom. Therefore we write

$$\Psi[g, \phi] = \Psi[g_{\text{in}}, \phi_{\text{in}}, g_{\text{out}}, \phi_{\text{out}}]. \tag{12}$$

This state can also be represented by a pure-state density matrix

$$\rho[g_{\text{in}}, \phi_{\text{in}}, g_{\text{out}}, \phi_{\text{out}}|g'_{\text{in}}, \phi'_{\text{in}}, g'_{\text{out}}, \phi'_{\text{out}}] = \Psi[g_{\text{in}}, \phi_{\text{in}}, g_{\text{out}}, \phi_{\text{out}}]\Psi^*[g'_{\text{in}}, \phi'_{\text{in}}, g'_{\text{out}}, \phi'_{\text{out}}]. \tag{13}$$

By tracing out over the inside degrees of freedom, we get the mixed-state density matrix

$$\rho_{\text{out}}[g_{\text{out}}, \phi_{\text{out}}|g'_{\text{out}}, \phi'_{\text{out}}] = \int \mathcal{D}g_{\text{in}}\, \mathcal{D}\phi_{\text{in}}\, \rho[g_{\text{in}}, \phi_{\text{in}}, g_{\text{out}}, \phi_{\text{out}}|g_{\text{in}}, \phi_{\text{in}}, g'_{\text{out}}, \phi'_{\text{out}}], \tag{14}$$

which describes information available to an outside observer. Next we identify a clock-time of an outside observer, so that we can write

$$\rho_{\text{out}}[g_{\text{out}}, \phi_{\text{out}}|g'_{\text{out}}, \phi'_{\text{out}}] = \rho_{\text{out}}[g_{\text{out}}, \tilde{\phi}_{\text{out}}, q_{\text{clock out}}|g'_{\text{out}}, \tilde{\phi}'_{\text{out}}, q'_{\text{clock out}}]. \tag{15}$$

Finally, by considering the clock-diagonal matrix elements $q_{\text{clock out}} = q'_{\text{clock out}} \equiv t$, we get an "evolving" outside density matrix

$$\rho_{\text{out}}[g_{\text{out}}, \tilde{\phi}_{\text{out}}|g'_{\text{out}}, \tilde{\phi}'_{\text{out}}](t) \equiv \rho_{\text{out}}[g_{\text{out}}, \tilde{\phi}_{\text{out}}, t|g'_{\text{out}}, \tilde{\phi}'_{\text{out}}, t]. \tag{16}$$

Clearly, the t-evolution described by (16) may not be unitary. At times t for which the black hole has evaporated completely, (16) may correspond to a mixed state, in accordance with predictions of the semiclassical theory [1]. One could think that it is merely a restatement of the information paradox, but it is actually much more than that. Unlike the standard statement of the paradox [1], such a restatement contains also a resolution of the paradox. Namely, from the construction of (16) it is evident that there is nothing fundamental about such a violation of unitarity. No information is really lost. The full information content is encoded in the pure state (13) equivalent to the wave function (12). This is very different from the information loss in the standard formulation [1], where information seems to be really lost and no description in terms of pure states seems possible.

To see more explicitly where the information is hidden, it is useful to introduce *two* clocks, such that (12) can be written as

$$\Psi[g_{\text{in}}, \phi_{\text{in}}, g_{\text{out}}, \phi_{\text{out}}] = \Psi[g_{\text{in}}, \tilde{\phi}_{\text{in}}, q_{\text{clock in}}, g_{\text{out}}, \tilde{\phi}_{\text{out}}, q_{\text{clock out}}]. \tag{17}$$

Here $q_{\text{clock in}}$ and $q_{\text{clock out}}$ are configuration variables describing an inside clock and an outside clock, respectively. Assuming that the black hole eventually evaporates completely, the inside clock cannot show a time larger than some value t_{evap} corresponding to the time needed for the complete evaporation. More precisely, the probability that $q_{\text{clock in}} > t_{\text{evap}}$ is vanishing, so

$$\Psi[g_{\text{in}}, \tilde{\phi}_{\text{in}}, q_{\text{clock in}}, g_{\text{out}}, \tilde{\phi}_{\text{out}}, q_{\text{clock out}}] = 0 \quad \text{for} \quad q_{\text{clock in}} > t_{\text{evap}}. \tag{18}$$

The existence of the wave function (17) implies that the system can be described by a pure state even after the complete evaporation. However, this description is trivial, because (18) says that the wave function has a vanishing value for $q_{\text{clock in}} > t_{\text{evap}}$. Still, even a nontrivial pure-state description for $q_{\text{clock out}} > t_{\text{evap}}$ is possible, provided that $q_{\text{clock in}}$ is restricted to the region $q_{\text{clock in}} < t_{\text{evap}}$. In this case (17) describes the correlations between the outside degrees of freedom after the complete evaporation and the inside degrees of freedom before the complete evaporation. In other words, if one asks where the information after the complete evaporation is hidden, then the answer is – *it is hidden in the past*. Of course, experimentalists cannot travel to the past, so information is lost for the experimentalists. Yet, this information loss is described by a pure state, so one does not need to use the Hawking formalism [1] in which a state evolves from a pure to a mixed state. By avoiding this formalism one avoids its pathologies [20] too, which may be viewed as the main advantage of our approach.

One might object that information hidden in the past is the same as information destruction, but it is not. The difference is subtle and essential for our approach, so let us explain it once again more carefully. Information hidden in the past and information destruction are the same for an observer who views the world as an entity that evolves with time t in (16). However, such a view of the world is emergent rather than fundamental, because time is emergent rather than fundamental. At the fundamental level there is no time and no evolution. The fundamental world is static and unitary, as described by (2). The concept of "past" refers to something which does not longer exist at the emergent level, but it still exists at the fundamental level. Thus, at the fundamental level, information is better described as being *present* in the past and only *hidden* for an emergent observer, rather than being destroyed. In this sense, our resolution of the information paradox does not remove the non-unitary time evolution entirely. Instead, *it shifts the non-unitary time evolution from a fundamental level to an emergent one*.

One might still argue that we have only shifted the problem (from one level to another) and not really solved it. But in our view such a shift of the problem is also a solution, or at least a crucial part of a solution. Namely, it is typical for emergent theories in physics that they lack full self-consistency, even when the underlying fundamental theories are self-consistent. Indeed, a presence of an inconsistency in an otherwise successfull physical theory is often a sign that this theory is not fundamental, but emergent. (A classic example is the ultraviolet catastrophe in classical statistical mechanics. It was resolved by Planck and others by recognizing that classical statistical mechanics emerges from more fundamental quantum statistical mechanics, which does not involve the ultraviolet catastrophe. In this way the inconsistency of classical statistical mechanics was not removed, but shifted from a fundamental to an emergent level.) In our case of the black-hole information paradox, the emergent theory is not self-consistent as it violates unitarity. We resolve the problem by identifying a more fundamental unitary theory from which the unitarity-violating theory emerges. The unitarity violation is nothing but a sign that *the emergent description in terms of time evolution is not fully applicable to the phenomenon of black-hole evaporation, and the fundamental theory involving no time evolution is a more ap-*

propriate description. In our opinion, it is a legitimate resolution of the black-hole information paradox, even if an unexpected one.

4 Conclusion

To conclude, canonical quantum gravity lacks a fundamental notion of time evolution, which implies trivial unitarity of the theory at the fundamental level. Time and evolution are emergent concepts, defined with the aid of a physical clock. In general, such a clock-time only has a local meaning and is represented by a quantum observable. Therefore, information present on a global spacelike hypersurface does not play any fundamental role. Consequently, even if observers living after the complete evaporation of a black hole cannot see all information encoded in the wave function of the universe, which can be interpreted as effective violation of unitarity for the observers, the full wave function of the universe still contains all the information and no fundamental violation of unitarity takes place. In this way both fundamental unitarity and phenomenological information loss may peacefully coexist, which resolves the black-hole information paradox.

Acknowledgements

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